Thesis for the degree of Licentiate of Engineering

Resource Allocation in Flexible-Grid Optical Networks with Nonlinear Interference

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To my family

Abstract

As the backbone of modern communications, the optical networks are anticipated to provide higher data rate and flexibility to support the exponentially growing volume and heterogeneity of traffic requirements. Flexible-grid optical networks have been proposed to improve the utilization of spectrum resources. However, the physical layer conditions are more complex and variable in flexible-grid networks than fixed-grid wavelength division multiplexing networks. Therefore, the consideration of physical layer impairment (PLI) is necessary in the planning stage of flexible-grid optical networks.

In this thesis, we mainly study the allocation of physical layer resources such as modulation formats, power spectral densities (PSDs), and carrier frequencies for all the channels in flexible-grid networks. To accurately estimate the quality of transmission, both linear and nonlinear PLIs are considered. Novel resource allocation problems are formulated and solved to include a nonlinear channel model and additional degrees of freedom in flexible-grid optical networks. The network performance and efficiency improvements over previous works are demonstrated via numerical calculations.

Paper A proposes a resource allocation algorithm for a single flexible-grid fiber link based on a nonlinear signal distortion model, i.e., the Gaussian noise (GN) model, as a first step towards a whole-network design. Based on the accurate estimation of channel PLIs provided by the GN model, the proposed algorithm can allocate resources more efficiently in terms of spectrum usage. Its performance is demonstrated through comparisons with the benchmark algorithm utilizing the transmission reach method.

Paper B and Paper C extend the proposed formulation to the network level and study the impact of nonlinear interference on transmission reaches and resource usage. Specifically, with precalculated routes and spectrum orderings for all the channels, Paper B allocates resources in a three-node network with optimized PSD. In Paper C, the algorithm is further extended to more complex networks to demonstrate its scalability and performance.

In Paper D, we jointly allocate route, spectrum, modulation format, and PSD for each connection request in the flexible-grid network. A mixed integer linear programming problem is formulated to search for the optimal resource allocation. A heuristic algorithm based on problem decomposition is also developed to reduce computational complexities. The joint resource allocation approach can improve the spectrum efficiency even further compared with previous separate planning strategies.

Keywords: Flexible-grid optical network, elastic optical network, resource allocation, physical layer impairment, Gaussian noise model, nonlinear interference

List of Included Publications

The thesis is based on the following appended papers:

[A] L. Yan, E. Agrell, H. Wymeersch, P. Johannisson, R. Di Taranto, and M. Brandt-Pearce, "Link-Level Resource Allocation for Flexible-Grid Nonlinear Fiber-Optic Communication Systems," in *IEEE Photonics Technology Letters*, vol. 27, no. 12, pp. 1250–1253, June 2015.

[B] L. Yan, E. Agrell, and H. Wymeersch, "Resource Allocation in Nonlinear Flexible-Grid Fiber-Optic Networks," in *Proceedings of Optical Fiber Communication Conference (OFC)*, paper Tu2I.5, Mar. 2015. (top scored)

[C] L. Yan, E. Agrell, H. Wymeersch, and M. Brandt-Pearce, "Resource Allocation for Flexible-Grid Optical Networks With Nonlinear Channel Model," in *Journal of Optical Communications and Networking*, vol. 7, no. 11, pp. B101–B108, Nov. 2015. (invited)

[D] L. Yan, J. Zhao, E. Agrell, and H. Wymeersch, "Power Optimization in Nonlinear Flexible-Grid Optical Networks," in *Proceedings of European Conference and Exhibition on Optical Communication (ECOC)*, paper P6.11, Sep. 2015.

Other contributions by the author (not included in this thesis):

[E] J. Zhao, L. Yan, H. Wymeersch, and E. Agrell, "Code Rate Optimization in Elastic Optical Networks," in *Proceedings of European Conference and Exhibition on Optical Communication (ECOC)*, paper We.3.5.1, Sep. 2015.

[F] N. Dharmaweera, J. Zhao, L. Yan, M. Karlsson, and E. Agrell, "Traffic-grooming and multipath-routing enabled impairment-aware elastic optical networks," submitted to *Journal of Optical Communications and Networking.* (accepted)

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Acronyms

ASE:	Amplified spontaneous noise
BER:	Bit error rate
BPSK:	Binary phase shift keying
EDFA:	Erbium-doped fiber amplifier
FEC:	Forward error correction
GN:	Gaussian noise
ILP:	Integer linear programming
MCI:	Multi-channel interference
MINLP:	Mixed integer nonlinear programming
NLI:	Nonlinear interference
NLS:	Nonlinear Schrödinger
PLI:	Physical layer impairments
PM:	Polarization multiplexing
PSD:	Power spectral density
QAM:	Quadrature amplitude modulation
QoT:	Quality of transmission
QPSK:	Quadrature phase shift keying
RSA:	Routing and spectrum assignment
RWA:	Routing and wavelength assignment
SCI:	Self-channel interference
SNR:	Signal to noise ratio
WDM:	Wavelength division multiplexing
XCI:	Cross-channel interference

Contents

Part I

Overview

Chapter 1

Introduction

1.1 Background and Motivation

Being the foundation of today's information society, the fiber-optical communication networks were first invented in the 1970s and since then gradually revolutionized the world by connecting everything easier and faster. Thanks to the extremely high bandwidth and long transmission distances, optical networks have been an enabling technology for Internet, data centers, and various communication networks. The enormous growth of these applications, drives the development of optical networks toward greater efficiency and flexibility [1]. Numerous techniques have been innovated in optical transmission systems to transmit data at higher rate over longer distance [2].

Nowadays, wavelength division multiplexing (WDM) is commonly used in optical networks to transmit 40-80 wavelengths on a fiber pair [3]. These wavelengths are located on predetermined wavelength grids, which divide the optical spectrum into fixed 50 GHz spectrum slots [4] carrying fixed data rates. Due to the exponentially growing network traffic demands, the data rate per wavelength is also increased from 10 Gbit/s to 100 Gbit/s and beyond [2]. However, current optical networks will not be able to support bit rates greater than 100 Gbit/s per wavelength because of the fixed spectrum grids. To properly address this challenge, more flexible networks are considered recently [3], which combine adaptive and intelligent hardware to enable new flexible-grid optical networks.

In flexible-grid optical networks, various bandwidths and data rates are enabled by adaptive transceivers [5]. In the spectrum domain, optical channels will be no longer situated in fixed spectrum slots, but rather at irregular frequencies based on their different data rates, transmission distances, and network conditions. Furthermore, the software-programmable optical transceivers allow us to adaptively change transmission parameters such as modulation format, carrier frequency, and power spectral density (PSD) for every channel to optimize the network globally [6].

When designing a network, we often need to balance the various trade-offs among communication requirements and constraints. Consequently, it is very important to allocate resources in flexible-grid optical networks sophisticatedly so that the full benefits of the technology could be attained. An efficient resource allocation algorithm is thus one critical component that enables optical networks to work in practice. In resource allocation algorithms, the routes and various transmission parameters need to be determined such that the data rate requirements of all the channels are fulfilled with acceptable bit error rates (BERs), or equivalently, satisfactory qualities of transmission (QoTs).

The resource allocation problem in flexible-grid optical networks is more complex than in traditional fixed-grid WDM networks due to the additional degrees of freedom and new network features. Furthermore, the heterogeneous distributions of wavelengths, powers, and bandwidths can incur high nonlinearities in optical fibers, which may significantly impair channel QoTs. Hence, it is necessary to have an accurate channel model to predict physical layer impairments (PLIs) and guarantee QoTs.

Traditionally, transmission reach methods have been used to estimate PLIs for fixed-grid WDM networks [7, 8]. The transmission reach of a specific modulation format is the worst-case distance that can be transmitted in a fully loaded spectrum. This approach usually overprovisions the signal-to-noise ratio (SNR) margins such that all the channels have acceptable QoTs in any network condition, which results in waste of optical fiber resources in most of the cases. Moreover, in flexible-grid optical networks, signals with various modulation formats and bandwidths can share the same link. The resulting PLIs are notably more state-dependent than in fixed-grid WDM networks [Paper A].

The Gaussian noise (GN) model has been proposed to characterize the nonlinear signal distortions in optical fibers with reasonable accuracy and low computational complexity [9–14]. Based on perturbation analysis, the GN model finds approximate analytical solutions to the nonlinear pulse propagation equation in dispersion uncompensated fiber, where the nonlinear interference (NLI) behaves as stationary additive Gaussian noise [12]. With the GN model, we can precisely predict SNRs, and thus QoTs of all the channels based on the allocated routes and transmission parameters.

1.2 Aim of the Thesis

The main aim of this thesis is to incorporate the GN model into resource allocation algorithms in flexible-grid optical networks. Specifically, we seek answers to the following questions:

- How should resource allocation problems in flexible-grid optical networks with the GN model be formulated?

- What is the impact of considering nonlinear impairments in flexible-grid optical networks?

- Is it beneficial to jointly optimize transmission parameters in the flexible-grid optical networks?

The transmission parameters are defined as modulation formats, carrier frequencies, and PSDs of optical channels in this thesis.

1.3 Thesis Outline

The structure of this thesis is organized as follows. Chapter 2 introduces the PLIs in an optical fiber. Various PLIs in the optical fiber and the analytical GN model will be introduced, followed by a discussion about the QoT. In Chapter 3, after reviewing common resource allocation formulations in traditional fixed-grid WDM networks, the transmission parameter optimization problem in flexible-grid networks with nonlinear interference is considered. Chapter 4 summarizes our contributions in the appended papers.

Chapter 2

The Fiber-Optical Channel

The optical fiber is a cylindrical waveguide composed of a dielectric core and cladding. The refractive index of the core is higher than the cladding such that light can be trapped in the fiber. A mode describes the spatial distribution of the electromagnetic field in optical fibers. In this thesis, we are concerned with single-mode fibers, which supports only one propagating mode and is commonly used for backbone optical networks.

The response of any dielectric to light becomes nonlinear for intense electromagnetic fields [15, p. 26]. This is the origin of nonlinearities in optical fibers. As the light propagates in the fiber for a long enough distance, the nonlinear effects will accumulate and distort signals modulated on the lightwave. The propagation of an electromagnetic field in optical fibers are governed by the Maxwell's equations and the boundary conditions imposed by the cylindrical dielectric core and cladding, which are further simplified to a pulse-propagation equation and referred to as the nonlinear Schrödinger (NLS) equation [15, pp. 25–41].

In addition to NLI, optical signal in fibers also suffers from various impairments such as power losses, chromatic dispersion, and amplified spontaneous emission (ASE) noise from erbium-doped fiber amplifiers (EDFAs). Unlike the interference introduced by the nonlinear effects, these impairments either scales linearly with the signal power or is a constant parameter of the fiber system.

It is not trivial to obtain the statistical properties of the noise in optical fibers. Specifically, the prediction of NLI needs to solve the NLS equation, a nonlinear partial differential equation that does not generally have analytic solutions. Fortunately, the GN model is recently proposed as an analytical approximation of the nonlinear signal distortion. We can use the GN model in network resource allocation algorithms to easily quantify the nonlinear impairments in the physical layer.

This chapter is organized as follows. In Section 2.1, starting with the NLS equation, the main PLIs in the fiber are described. The GN model will be introduced along with its associating assumptions in Section 2.2. The computation of channel SNRs, which is a measurement of QoTs based on the GN model and expressions of other PLIs, will be described in Section 2.3.

2.1 Pulse Propagation in Optical Fibers

The single-polarization pulse-propagation equation in an optical fiber is modeled using the NLS equation with loss [15, p. 40]

$$\frac{\partial A}{\partial z} = i\gamma|A|^2 A - i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2}A, \tag{2.1}$$

where $A = A(z,t)$ is the slowly varying complex envelope of the optical field, z is the distance of propagation, γ is the nonlinear coefficient of the fiber, $|A|^2$ is the power of the optical envelope, β_2 is the group velocity dispersion coefficient, α is the power attenuation factor of the fiber, and t is the time coordinate in a reference frame moving with the optical envelope. Given an initial envelope $A(0,t)$ at the input of a fiber, (2.1) tells us the propagated pulse envelope at any distance z in the fiber. The Manakov equation is the double-polarization counterpart of the NLS equation.

The three terms at the right-hand side of (2.1) govern, respectively, the effects of nonlinearity, dispersion, and fiber losses on optical pulses. In the following sections, we will introduce these impairments in detail.

2.1.1 Chromatic Dispersion

In a dispersive medium the group velocities of different wavelengths are different. This phenomenon is called group velocity dispersion or chromatic dispersion. In long haul optical communication systems, the accumulated chromatic dispersion is large enough to broaden the pulse width in the time domain and thus leads to inter-symbol interference.

To study the pulse propagating in a linear dispersive medium, we can neglect the nonlinearity and power losses by setting the nonlinear coefficient $\gamma = 0$ and the power attenuation factor $\alpha = 0$. The NLS equation thus becomes

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}. \tag{2.2}$$

The closed-form solution of (2.2) in the spectrum domain is

$$\tilde{A}(z,\omega) = \tilde{A}(0,\omega)\exp(i\beta_2\omega^2 z/2), \tag{2.3}$$

where $\tilde{A}(z,\omega)$ is the Fourier transform of the time domain pulse envelope $A(z,t)$. From the solution of (2.2) we know that the chromatic dispersion can be modeled as a linear all-pass filter. The amplitude of the pulse frequency is unaffected, but a frequency-dependent phase shift is introduced.

The chromatic dispersion also results in chirped optical pulses, i.e., the instantaneous frequency changes at different time coordinates of the pulse envelope. This is because different frequency components travel at different speeds. After the propagation, the high velocity component appears at the leading edge of the pulse, whereas the low velocity component is delayed at the trailing edge.

Traditionally, chromatic dispersion is compensated for in the optical domain by a dispersion-compensating fiber or a fiber Bragg grating. The dispersion-compensating

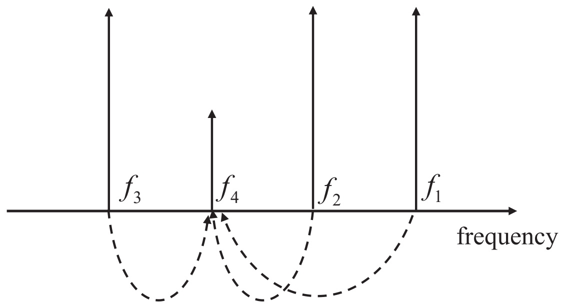

Figure 2.1: Four-wave mixing generates new frequency component f_4 from three existing frequencies f_1, f_2 and f_3.

fiber has the opposite sign of β_2 compared to the single-mode fiber. The fiber Bragg grating reflects different frequency components of the chirped optical pulse at different positions in the fiber grating to compress the pulses.

In modern high speed coherent transmission systems, chromatic dispersion is compensated by digital filters in the receiver. This digital signal processing technique avoids additional nonlinearities or losses induced by the optical compensation modules, and enables more flexible deployment of fibers. In the rest of this thesis, we will assume that the chromatic dispersion is compensated through the digital signal processing at the receiver side and no optical dispersion compensation is present in the fiber link.

2.1.2 Nonlinear Interference

The nonlinear effects in an optical fiber originate from nonlinear refraction, a phenomenon referring to the intensity dependence of the refractive index. In the presence of the optical field, the refractive index of the fiber is slightly changed. And this varied refractive index further changes the phase of optical signal and results in all kinds of NLI. Specifically, the fiber nonlinearity can be understood by ignoring the dispersion and power loss in (2.1) as

$$\frac{\partial A}{\partial z} = i\gamma |A|^2 A. \tag{2.4}$$

The solution of (2.4) is

$$A(z,t) = A(0,t) \exp\left[i\gamma |A(0,t)|^2 z\right]. \tag{2.5}$$

Equation (2.5) describes the nonlinear distortion in the presence of one pulse modulated on a single wavelength, where the signal phase is changed proportionally to the signal power $|A(0,t)|^2$. In WDM and flexible-grid optical networks, signals from multiple channels with different wavelengths may overlap in the time domain and lead to more complex interference like four-wave mixing, where interactions between three existing

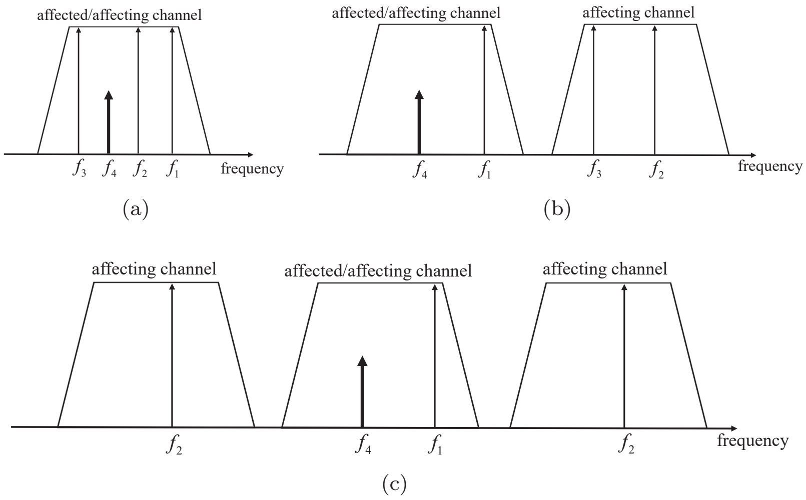

Figure 2.2: Classification of NLI. Thin tall arrows (f_1, f_2, and f_3): generating frequency components. Thick short arrow (f_4): generated frequency component. (a): SCI, channel is affected by itself; (b): XCI, channel is affected by the interaction between itself and another channel; (c): MCI, at least two affecting channels outside the affected channel are involved.

wavelengths produce a fourth wavelength [15, pp. 369–370]. An illustration of four-wave mixing is shown in Figure 2.1.

From a spectrum domain perspective, the nonlinear effects can be classified into three categories according to their origins: self-channel interference (SCI), cross-channel interference (XCI), and multi-channel interference (MCI). As is shown in Figure 2.2, for a specific channel, its SCI is the interference generated by the channel to itself. The XCI of the channel comes from its interaction with another channel. Its MCI is produced by at least two other channels outside the affected channel. In a dispersion uncompensated transmission system, SCI and XCI are the dominant NLI, while the relative strength of MCI is almost always negligible [12].

Unlike the chromatic dispersion that is equivalent to a linear all-pass filter, the nonlinear effects generate new frequencies in the spectrum domain. The new frequency component will interfere with other existing wavelengths and degrade QoTs of copropagating channels. Moreover, the nonlinear effects can further interact with chromatic dispersion and ASE noise to generate complicated nonlinear phase and amplitude noise, which is hard to capture analytically. Actually, nonlinearity is a serious problem limiting the efficiency of optical transmission systems.

Nonlinearities can be partly compensated for by digital backpropagation, which solves the inverse NLS equation by simulating the received signal with inversed parameters $(-\beta_2, -\gamma, -\alpha)$ [16–18]. In this thesis, we assume that the SCI is compensated for by digital backpropagation for some of the resource allocation algorithms [Paper B, Paper

C]. These algorithms can be easily extended to nonlinearity uncompensated scenarios by modifying the NLI estimation.

2.1.3 Fiber Loss and ASE Noise

The power loss of optical signals inside a fiber comes from sources such as Rayleigh scattering, material absorption, bending of the fiber, and scattering of light at the core-cladding interface [15, pp. 5–6]. The fiber loss can be described mathematically by ignoring the dispersion and nonlinearity in (2.1) as

$$\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A. \tag{2.6}$$

The solution of (2.6) is

$$A(z,t) = A(0,t) \exp\left[-\alpha z/2\right]. \tag{2.7}$$

Equation (2.7) shows that the pulse amplitude decreases exponentially as a function of the propagation distance z. If $P_0 = \int_{-\infty}^{\infty} |A(0,t)|^2 dt$ is the power of the optical signal at the input of a fiber with length of L, the received power P_R is given by

$$P_R = P_0 \exp(-\alpha L). \tag{2.8}$$

The attenuation constant α is a measure of total fiber losses in linear scale. It is customary to express α in units of dB/km using the relation

$$\alpha_{\text{dB}} = -\frac{10}{L} \log\left(\frac{P_R}{P_0}\right) = 4.343\alpha. \tag{2.9}$$

In the commercial optical communication systems, fibers usually exhibit a loss around 0.2 dB/km, which means that after the transmission of a fiber span (usually around 100 km) the optical power will be attenuated by more than 20 dB and well below the sensitivity threshold of photo-detectors in the receiver. Consequently, in the long-haul transmission, amplifiers are necessary to boost the intensity of optical signals periodically at the end of fiber spans. The EDFA is the most commonly used amplifier in the optical communication systems.

Inevitably, amplifiers will also induce ASE noise to optical signals. The ASE noise is intrinsic to EDFAs [19, pp. 255–256] and can be modeled as additive white Gaussian noise with the PSD per span per polarization

$$G_{\text{span}}^{\text{ASE}} = (e^{\alpha L} - 1) h\nu n_{\text{sp}}, \tag{2.10}$$

where h is the Planck constant, ν is the light frequency, and n_{sp} is the spontaneous emission factor. The ASE noise can interact with other impairments in the fiber to deteriorate the signal quality.

Note that commercial EDFAs usually have a flat gain spectrum such that different wavelengths can experience the same gain [19, pp. 258–259]. Consequently, as long as the EDFA is not saturated, channels with different input PSDs will keep the PSD difference (in dB) after the amplification.

2.2 Gaussian Noise Model

In Section 2.1, three phenomena of the pulse propagation in an optical fiber are introduced, i.e., the chromatic dispersion, nonlinear effect, and ASE noise induced by EDFAs. Among these PLIs, the chromatic dispersion can be totally compensated by the digital signal processing module at the receiver side. Consequently, in the future, optical dispersion compensation hardware will be completely removed from fiber links to enable more flexible transmission. As an additional benefit, the signal propagation and nonlinearity generation in the fiber are also tremendously simplified in dispersion uncompensated transmission. Analytical nonlinear propagation models can be used for performance prediction, and the NLI manifests itself as additive Gaussian noise due to the unmitigated dispersion effect [9–14].

The Gaussian noise (GN) model has been proposed to predict the NLI in dispersion uncompensated fiber links. The GN model holds under the following assumptions: 1) a coherent polarization-multiplexed (PM) system without optical compensation for chromatic dispersion; 2) for each channel, the transmitted signal PSD is equal in both polarizations; 3) each channel spectrum is rectangular and does not overlap with neighboring channels; 4) the NLI generated in different fiber spans sum incoherently over the whole link; 5) the losses of each fiber span are compensated by EDFA at the end of the span; 6) all spans are long enough to have the same effective length; 7) the MCI is neglected due to its negligible strength.

Under the above-mentioned assumptions, by applying perturbation analysis and approximations to the pulse-propagation equation, the NLI PSD per span per polarization for channel i can be expressed as [9]

$$G_{i,\mathrm{span}}^{\mathrm{NLI}} = G_{i,\mathrm{span}}^{\mathrm{SCI}} + G_{i,\mathrm{span}}^{\mathrm{XCI}} \tag{2.11}$$

where

$$G_{i,\mathrm{span}}^{\mathrm{SCI}} = \frac{3\gamma^2}{\alpha^2} F_{ii}^2 G_i^3, \quad G_{i,\mathrm{span}}^{\mathrm{XCI}} = \frac{6\gamma^2}{\alpha^2} G_i \sum_{\substack{j\in D\\ j\neq i}} F_{ij}^2 G_j^2. \tag{2.12}$$

Here D is the set of channels in the network. $G_{i,\mathrm{span}}^{\mathrm{SCI}}$ and $G_{i,\mathrm{span}}^{\mathrm{XCI}}$ are the SCI and XCI PSDs per span per polarization of channel i, respectively. G_i is the transmitted PSD per polarization for channel i. F_{ij}^2 for $i, j \in D$ is the NLI efficiency function representing the NLI strength:

$$\begin{aligned} F_{ij}^2 = & \frac{2}{\xi}\Bigg\{ \mathrm{Im}\,\mathrm{Li}_2\left[\sqrt{-1}\frac{\Delta f_i}{2}\left(f_i - f_j + \frac{\Delta f_j}{2}\right)\xi\right] \\ & + \mathrm{Im}\,\mathrm{Li}_2\left[\sqrt{-1}\frac{\Delta f_i}{2}\left(f_j - f_i + \frac{\Delta f_j}{2}\right)\xi\right]\Bigg\}, \end{aligned} \tag{2.13}$$

with

$$\xi = \frac{4\pi^2|\beta_2|}{\alpha}, \tag{2.14}$$

and Li_2 is the dilog function. f_i and Δf_i are the carrier frequency and bandwidth of channel i for $i \in D$, respectively.

The dilog function in (2.13) can be simplified using its asymptotic expansion. As a result, the NLI efficiency function F_{ij}^2 for $i, j \in D$ can be approximated by the hyperbolic arcsin function if $i = j$ [14], and the logarithm function if $i \neq j$ [9]

$$F_{ii}^2 \approx \frac{\alpha}{2\pi|\beta_2|} \operatorname{arcsinh}\left(\frac{\pi^2|\beta_2|}{2\alpha}\Delta f_i^2\right),$$
$$F_{ij}^2 \approx \frac{\alpha}{4\pi|\beta_2|} \log\left|\frac{|f_i - f_j| + \Delta f_j/2}{|f_i - f_j| - \Delta f_j/2}\right|, i \neq j. \tag{2.15}$$

The hyperbolic arcsin function is used for the SCI coefficient F_{ii}^2 as it approximates the dilog function better than the logarithm function when $\pi^2|\beta_2|\Delta f_i^2/2\alpha \ll 1$, in which case the logarithm function tends toward negative infinity. On the contrary, the argument of the XCI coefficient F_{ij}^2 in (2.15) is always greater than 1, and thus the logarithm function gives acceptable approximation error.

From (2.11), (2.12), and (2.15), the NLI PSD per span per polarization is

$$\begin{aligned} G_{i,\text{span}}^{\text{NLI}} =& \frac{3\gamma^2}{2\pi\alpha|\beta_2|} G_i \Bigg(G_i^2 \operatorname{arcsinh}\left(\frac{\pi^2|\beta_2|}{2\alpha}\Delta f_i^2\right) \\ &+ \sum_{\substack{j\in D\\ j\neq i}} G_j^2 \log\left|\frac{|f_i - f_j| + \Delta f_j/2}{|f_i - f_j| - \Delta f_j/2}\right| \Bigg). \end{aligned} \tag{2.16}$$

Since we have assumed that NLI from different spans are accumulated incoherently, (2.16) can be readily extended to a network of multiple spans and links as [Paper C]

$$\begin{aligned} G_i^{\text{NLI}} =& \frac{3\gamma^2}{2\pi\alpha|\beta_2|} G_i \Bigg(G_i^2 N_i^{\text{s}} \operatorname{arcsinh}\left(\frac{\pi^2|\beta_2|}{2\alpha}\Delta f_i^2\right) \\ &+ \sum_{\substack{j\in D\\ j\neq i}} G_j^2 N_{ij}^{\text{s}} \log\left|\frac{|f_i - f_j| + \Delta f_j/2}{|f_i - f_j| - \Delta f_j/2}\right| \Bigg), \end{aligned} \tag{2.17}$$

where N_i^{s} is the number of spans transversed by channel i, and N_{ij}^{s} is the number of channels shared by channel i and j. N_{ij}^{s} equals 0 if channel i and j do not share any link. Notice that the NLI PSD is independent of fiber span length since we have assumed that all spans are long enough to have the same effective length.

It follows from (2.17) that the NLI PSD of one channel depends on the routes and transmission parameters of all the channels in optical networks. Therefore, a global knowledge of all the resources allocated in the network is required to estimate the NLI, implying a centralized network planning algorithm [9].

2.3 Quality of Transmission

The purpose of the GN model is to estimate QoTs, which is specified by the BER of each channel at the receiver side after forward error correction (FEC) decoding (i.e., post-FEC BER). In this thesis, we assume hard decision FEC decoding and fixed code type and code rate for all the channels in the network. Therefore, the post-FEC BER can be calculated given the BER before FEC decoder (i.e., pre-FEC BER), which is further related to the SNR of the received signal. To guarantee a satisfactory QoT, the actual SNR should be no less than a certain SNR threshold that is determined by the selected modulation format and FEC code type and code rate. On the other hand, the actual SNR of channel i can be estimated by

$$\mathrm{SNR}_i = \frac{G_i}{G_i^{\mathrm{ASE}} + G_i^{\mathrm{NLI}}}, \tag{2.18}$$

where G_i is the transmitted PSD per polarization, $G_i^{\mathrm{ASE}} = G_{\mathrm{span}}^{\mathrm{ASE}} N_i^{\mathrm{s}}$ is the accumulated ASE noise along the fiber links transversed by channel i, $G_{\mathrm{span}}^{\mathrm{ASE}}$ is the ASE noise per span per polarization in (2.10), and G_i^{NLI} is the NLI PSD given by the GN model in (2.17). The requirement for acceptable QoT is hence translated to an inequality constraint on SNR

$$\mathrm{SNR}_i \geq \mathrm{SNR}_{i,\mathrm{th}}, \tag{2.19}$$

where $\mathrm{SNR}_{i,\mathrm{th}}$ is the SNR threshold of channel i.

According to (2.19), the SNR threshold is the minimum SNR achieving certain pre-FEC BER requirement. The set of available modulation formats is PM binary phase shift keying (BPSK), PM-quadrature phase keying shift (QPSK), PM-8 quadrature amplitude modulation (QAM), PM-16QAM, PM-32QAM, and PM-64QAM. The spectral efficiencies (SEs) are denoted by c and listed in Table 2.1. Since the modulation formats in Table 2.1 have distinct SEs, we can use c to denote them as well. The SNR thresholds of these modulation formats with Gray mapping [20, 21], $\mathrm{SNR}_{\mathrm{th}}(c)$, are also listed in Table 2.1. Note that it is possible to use a subset of the available modulation formats, and the pre-FEC BER requirement may change according to specific requirement[1].

Table 2.1: The available modulation formats, their SEs, and linear scale SNR thresholds $\mathrm{SNR}_{\mathrm{th}}(c)$ to achieve a pre-FEC BER of 4×10^{-3}.

Modulation Format	c (bit/s/Hz)	$\mathrm{SNR}_{\mathrm{th}}(c)$
PM-BPSK	2	3.52
PM-QPSK	4	7.03
PM-8QAM	6	17.59
PM-16QAM	8	32.60
PM-32QAM	10	64.91
PM-64QAM	12	127.51

[1] Here, the pre-FEC BER threshold of 4×10^{-3} is chosen to be the same as the benchmark [22] of our proposed resource allocation algorithms to facilitate the numerical comparisons [Paper B, Paper C].

Chapter 3

Network Resource Allocation

In today's optical networks, a number of distinct wavelengths are used to implement separate channels [23]. This technology is called wavelength routing, where the traffic is carried by logical connections from sources to destinations. In addition, most of the optical networks are transparent, i.e., all the channels are purely in the optical domain and no wavelength conversions are used. In transparent networks, the channels are transmitted through the network without going through electronic routers or optical-electrical-optical regenerations. Wavelength routed transparent networks are a cost-effective solution for transport networks. In this thesis, we will assume that the optical networks are transparent and all the connections are wavelength routed.

In wavelength routed networks, a channel consists not only the transmission parameters such as modulation formats, carrier frequencies, and launched channel PSDs, but also a set of fiber links connecting the source and destination nodes. As a result, in addition to the configuration of transmission parameters, we also need to assign links to form a path in the network level resource allocation problem. In traditional fixed-grid WDM networks, this network level planning is called the routing and wavelength assignment (RWA) problem. In flexible-grid optical networks, however, connection requests may have various data rate requirements, resulting in different channel bandwidths. In this context, a fraction of spectrum instead of a single wavelength should be allocated to one connection request. Moreover, due to the flexibility introduced by new network features, transmission parameters need to be determined for channels as well. To address these issues, new resource allocation algorithms have to be developed.

This chapter is organized as follows. In Section 3.1, the problem statement of the RWA in fixed-grid WDM networks along with common mathematical formulations are introduced. In Section 3.2 we discuss the extension of the RWA to flexible-grid optical networks, which is decomposed into simpler subproblems and solved suboptimally.

3.1 Routing and Wavelength Assignment in WDM Networks

The main focus of the WDM network planning is to establish channels according to connection requests. A channel consists of a set of fiber links connecting the source and destination nodes, and a particular wavelength on each of these links for the channel [23]. The RWA problem thus deals with the selection of routes and wavelengths. A feasible routing and wavelength assignment must satisfy two constraints.

- *Wavelength continuity constraint.* The same wavelength must be assigned to all the links transversed by a channel.

- *Distinct wavelength constraint.* If two or more channels share a common link, each must be assigned a distinct wavelength.

Hence, the RWA problem can be stated in terms of input, output, constraint, and objective as follows:

- *Input:* the network topology and a set of connection requests

- *Output:* the routes and wavelengths assigned to each connection request

- *Constraint:*

    1. wavelength continuity constraint
    2. distinct wavelength constraint

- *Objective:* minimize the number of wavelengths used to establish all the connection requests

Note that in fixed-grid WDM networks, the QoT is usually guaranteed by the transmission reach, which is the worst-case distance allowed to transmit with a specific modulation format in fully loaded fiber links. The transmission parameters such as channel PSDs and modulation formats are implicitly determined according to the transmission reaches and will not be concerned in the RWA problem.

In the optimization formulations of the resource allocation problems introduced in Section 3.1 and Section 3.2, we will use (V, E) to denote the network, where V is the set of nodes and E is the set of links. Each link has two fibers with opposite directions and multiple spans. The set of directional links going out from node n is denoted as E_n^+, and the set of directional links coming into node n is denoted as E_n^-. Note that E_n^+ and E_n^- are different sets of links with opposite directions. D is used to denote the set of channels that carry connection requests. W is the set of wavelengths on the fiber links. N is the number of nodes in the network.

The connection requests are denoted as an $N \times N$ traffic matrix $T = [t_{sd}]$, where t_{sd} is a non-negative integer representing the number of wavelengths that should be set up from node s to d[1]. The traffic matrix is assumed a static and deterministic. This is because the design of optical networks is accomplished by forecasting a certain traffic matrix every half a year or so [25, p. 598]. During this period, the traffic prediction is reasonably accurate and the variation of actual traffic demands is relatively slow. These variations can be handled by reconfigurable optical add-drop multiplexers installed in the network.

According to the constraints and objective listed above, we can obtain an integer linear programming (ILP) formulation to solve the problem optimally [26]. However, the RWA problem is NP-hard [27], so there exists no efficient solutions. The optimal solutions of the RWA problem can be achieved only for relatively simple network topologies with few connection requests, whereas most of the methods offer close-to-optimal solutions through heuristic algorithms. In the following sections, we will introduce a commonly used ILP formulation [26, 28] and its corresponding heuristic algorithm.

3.1.1 ILP Formulation

We will use l as link index, w as wavelength index, and i, j, and n as node index. The sets of decision variables in the ILP formulation are defined as follows:

- $c_{ij}^{lw} \in \{0, 1\}$: binary variable that indicates whether a channel from node i to j is established using wavelength w on link l
- $u^w \in \{0, 1\}$: binary variable that indicates whether wavelength w is used anywhere in the network
- ω_{total}: integer variable that indicates the maximum wavelength index used in the network

The RWA ILP formulation can be expressed as [26]:

$$\underset{c_{ij}^{lw}, u^w, \omega_{\text{total}}}{\text{minimize}} \quad \omega_{\text{total}} \tag{3.1}$$

$$\text{subject to} \quad \sum_{w \in W} \left(\sum_{l \in E_n^+} c_{ij}^{lw} - \sum_{l \in E_n^-} c_{ij}^{lw} \right) = \begin{cases} 0, & n \neq i, j; \\ t_{ij}, & n = i; \\ -t_{ij}, & n = j. \end{cases} \quad \forall n, i, j \in V, i \neq j \tag{3.2}$$

$$\sum_{\substack{i,j \in V \\ i \neq j}} c_{ij}^{lw} \leq 1, \quad \forall l \in E, \forall w \in W \tag{3.3}$$

$$\sum_{\substack{i,j \in V \\ i \neq j}} \sum_{l} c_{ij}^{lw} \leq u^w N(N-1)|E|, \quad \forall w \in W \tag{3.4}$$

$$\omega_{\text{total}} \geq w u^w, \quad \forall w \in W \tag{3.5}$$

[1] In [Paper D], to facilitate the comparison with the benchmark algorithm [24], we assume that the traffic matrix is symmetric, i.e., $t_{sd} = t_{ds}$. This assumption can reduce the computational complexity in the performance evaluation of algorithms. However, it is not necessarily true in practical network design.

Equation (3.2) is the multicommodity flow constraint at node n. Specifically, the first sum at the left hand side is the traffic going out of node n and the second sum represents the traffic coming into node n. If node n is an intermediate node on the path from source i to destination j, the traffic in should equal to the traffic out, since no traffic is added or dropped at node n. On the other hand, if node n is the source node i, then the outgoing traffic is equal to t_{ij} whereas the incoming traffic is zero. Similarly, if node n is the destination node j, the incoming traffic is t_{ij} and the outgoing flow is zero. Constraint (3.2) ensures that all the traffic demands are satisfied. In addition, by imposing no adding or dropping flows at intermediate nodes, the wavelength continuity constraints are implicitly ensured as the same wavelength will be used on all the links along the path. Inequality (3.3) represents the distinct wavelength constraint such that no two connections share the same wavelength on one link. Expression (3.4) ensures $u^w = 1$ when wavelength w is used on any link by any connection. Expression (3.5) calculates the highest wavelength index used by all the links.

The complexity of the ILP formulation is related to the number of binary variables and constraints. The number of variables c_{ij}^{lw} is equal to $N(N-1)|E||W|$, where $|W|$ is the number of wavelengths in one link and $|E|$ is the number of links in the network. There are also $|W|$ variables u^w and one variable ω_{total}. Hence, the total number of variables is $O(N^2|E||W|)$. For the number of constraints, equation (3.2) has N^3 constraints, and inequality (3.3) corresponds to $|E||W|$ constraints. For expressions (3.4) and (3.5), each of them consists of $|W|$ constraints. As a result, the number of constraints is $O(N^3)$, which is dominated by the multicommodity flow constraint (3.2).

The ILP formulation searches all the possible combinations of links to form paths for every connection. Therefore, the optimal solution is guaranteed. On the other hand, however, the ILP formulation is also highly symmetric. In the fixed-grid WDM network, all the channels are located in predefined uniform spectrum slots. As a result, if we exchange all the channels operating in one wavelength with channels in another wavelength, the highest wavelength index ω_{total} will not be changed, and the wavelength continuity and distinct constraints are still satisfied [29]. The symmetric property of the ILP formulation results in a large number of solutions yielding the same objective. The ILP solver has to explore the redundant solutions before discovering better ones, and the scalability of the ILP formulation is thus limited severely.

3.1.2 Heuristic RWA Algorithm

Because of the symmetry issue, it is not possible to obtain the optimal solution of the RWA ILP formulation for realistic optical networks within reasonable time. A large number of heuristic algorithms have been developed to solve the RWA problem suboptimally [23]. These algorithms usually utilize specific features of the problem to find approximate solutions.

In the following, we will describe one of such methods that decomposes the original RWA into two simpler subproblems and solve them sequentially [30]. The overall complexity of the two subproblems is much less than the original RWA problem. The heuristic first finds appropriate paths for every connection request in the *routing subproblem*, and then allocates wavelengths to paths in the *wavelength assignment subproblem*.

The routing subproblem can be stated as follows:

- *Input:* network topology and traffic matrix
- *Output:* set of paths Q_{ij} for the connection request from node i to j, where $i, j \in V$ and $i \neq j$
- *Constraint:* the traffic demands are satisfied
- *Objective:* minimize the maximum number of paths among all the links

In the routing subproblem, the wavelength assignment is neglected. This is equivalent to having wavelength converters with unlimited capability at every node in the network. According to the problem statement, we can formulate a low complexity ILP to solve the routing subproblem. Let us first define sets of parameters used in the routing ILP formulation:

- k: number of candidate paths precalculated for every pair of nodes before formulating the routing ILP, usually $k = 12$ [31] is sufficient to obtain close-to-optimal paths[2]
- P_{ij}: set of k candidate paths for connection requests from node i to j, which can be obtained using the k-shortest path algorithm [32]
- P_{ij}^l: set of candidate paths in P_{ij} using a particular link l

The variables involved in the ILP formulation are:

- $x_{ij}^p \in \{0, 1\}$: binary variable that indicates whether a candidate path $p \in P_{ij}$ is selected
- y_l: integer variable that indicates the number of selected paths using link l
- Y: integer variable that indicates the maximum number of paths among all the links, i.e., $Y = \max_l y_l$

The routing ILP formulation can be expressed as [33]:

$$\underset{x_{ij}^p, y_l, Y}{\text{minimize}} \quad Y \tag{3.6}$$

$$\text{subject to} \quad \sum_{p \in P_{ij}} x_{ij}^p = t_{ij}, \qquad \forall i, j \in V, i \neq j \tag{3.7}$$

$$y_l = \sum_{\substack{i,j \in V \\ i \neq j}} \sum_{p \in P_{ij}^l} x_{ij}^p, \qquad \forall l \in E \tag{3.8}$$

$$Y \geq y_l, \qquad \forall l \in E \tag{3.9}$$

[2]Note that if multiple wavelengths are requested by one node pair, all these wavelengths will be assigned to the same path. So the value of k is not dependent on the traffic matrix.

Expression (3.7) ensures that the connection request from node i to j is satisfied. Equation (3.8) calculates the number of selected paths passing through link l, and inequality (3.9) defines the maximum number of paths among all the links as $Y = \max_{l \in E} y_l$.

The number of variables x_{ij}^p is $N(N-1)k$, the number of variables y_l is $|E|$, and there is one variable Y. As a result, the number of variables in the routing ILP formulation is $O(N^2k)$. The number of equalities (3.7) is $N(N-1)$, the number of equalities (3.8) is $|E|$, and there are $|E|$ inequalities (3.9). Hence the number of constraints is $O(N^2)$. Obviously, the routing ILP formulation is much simpler than the original RWA ILP formulation in Section 3.1.1.

The final output Q_{ij} is the set of routes selected from P_{ij}, i.e., $Q_{ij} = \{p \in P_{ij} | x_{ij}^p = 1\}$ for $i, j \in V, i \neq j$. After obtaining the routes for each of the connection requests, we can formulate another ILP to solve the wavelength assignment subproblem, which can be stated as follows:

- *Input:* network topology, traffic matrix, and sets of paths Q_{ij} selected by the routing subproblem, where $i, j \in V$ and $i \neq j$
- *Output:* the wavelengths assigned to all the routes
- *Constraints:*
    1. wavelength continuity constraint
    2. distinct wavelength constraint
- *Objective:* minimize the number of wavelengths used to establish all the connection requests

The parameters used in the wavelength assignment ILP formulation are

- Q_{ij}^l: set of paths from node i to j using link l, $Q_{ij}^l \in Q_{ij}$, where $i, j \in V, i \neq j$, and $l \in E$

The variables in the ILP formulation are

- ω_{total}: integer variable that indicates the highest wavelength index used by any connection in the network
- $x_{ij}^{pw} \in \{0, 1\}$: binary variable that indicates whether a path $p \in Q_{ij}$ uses wavelength $w \in W$

The wavelength assignment ILP can be formulated as [30]:

$$\underset{x_{ij}^{pw}, \omega_{\text{total}}}{\text{minimize}} \quad \omega_{\text{total}} \tag{3.10}$$

$$\text{subject to} \quad \sum_{w} x_{ij}^{pw} = 1, \qquad \forall i, j \in V, i \neq j, \forall p \in Q_{ij} \tag{3.11}$$

$$\sum_{\substack{i,j \in V \\ i \neq j}} \sum_{p \in Q_{ij}^l} x_{ij}^{pw} \leq 1, \qquad \forall l \in E, \forall w \tag{3.12}$$

Expression (3.11) ensures that every path is assigned one wavelength. It also implicitly imposes the wavelength continuity constraint to all the paths. Inequality (3.12) guarantees that multiple paths sharing one common link must use different wavelengths, i.e., the distinct wavelength constraint.

The number of variables x_{ij}^{pw} is $R|W|$, where R is the total number of connection requests in the network, and there is one variable ω_{total}. So the number of variables in the ILP formulation is $O(R|W|)$. The number of constraints (3.11) is R, and the number of constraints (3.12) is $|E||W|$, so the total number of constraints is $O(|E||W| + R)$. In optical networks, the number of connection requests R usually has the same order of magnitude as N^2. Consequently, the number of variables is $O(N^2|W|)$ and the number of constraints is $O(|E||W| + N^2)$. Compared with the original RWA ILP formulation, which has $O(N^3|E||W|)$ variables and $O(N^3|W|)$ constraints, the wavelength assignment ILP formulation is simplified significantly. Furthermore, in practice, column generation [34] and valid inequality generation [35] techniques can be applied to the wavelength assignment ILP formulation to accelerate the solving effectively.

3.2 Resource Allocation in Flexible-Grid Networks

The flexible-grid optical networks can provide bandwidth-variable and highly spectrum-efficient modulation formats with fine granularity down to 3.125 GHz [3]. In this context, a connection with a large data rate demand would require multiple contiguous spectrum slots for transmission. Consequently, the traditional wavelength continuity and distinct wavelength constraints are transformed to the spectrum continuity and nonoverlapping spectrum constraints, respectively [33]. Examples of other new features introduced to flexible-grid optical networks are variable modulation formats and channel PSDs [Paper D] Hence the designer of networks need to allocate resources appropriately such that the benefits of new network features can be harvested.

The resource allocation problem in flexible-grid networks is divided into two parts: the routing and spectrum assignment (RSA) problem, and the optimization of transmission parameters, which include modulation formats, PSDs, and carrier frequencies in this thesis. The two parts can be solved jointly to yield close-to-optimal solutions [Paper D] at the cost of high computational complexity. This is because a large number of integer decision variables is involved in the optimization formulation of the joint resource allocation problem. These subproblems can also be solved separately to avoid the complexity issue. Note that both subproblems need to provide close-to-optimal solutions in order to guarantee a good solution to the whole problem. We will describe these subproblems in more detail in the following sections.

3.2.1 RSA Problem

The RSA problem is an extension of the RWA problem to include finer granular spectrum slots. Similar to the RWA problem, a feasible solution to the RSA problem has the following constraints on the path and spectrum.

- *Spectrum continuity constraint*. The same spectrum slots must be assigned to all the links transversed by a channel.

- *Spectrum contiguity constraint*. Contiguous spectrum slots should be assigned to one path.

- *Nonoverlapping spectrum constraint*. If two or more channels share a common link, their spectrum slots should be distinct and nonoverlapping.

The purpose of the spectrum contiguity constraint is to reduce the number of transponders used in the network. In the RWA problem, each path is assigned a single wavelength, and all the channels are located in fixed spectrum slots. As a result, the paths using one wavelength will not interfere with the choice of paths at other wavelengths. In the RSA problem, however, one path can occupy multiple spectrum slots depending on the data rate and modulation format of the connection request. Although the requested data rates are known from the traffic matrix, the modulation formats is still unknown at the RSA stage. Moreover, the traffic matrix is not uniform and channels with different bandwidths may coexist in the network. Hence, the selections of path and spectrum are coupled with each other tightly, which adds huge complexity to the RSA problem. Recall that the RWA problem is very hard to solve and we have to rely on heuristic algorithms to search for suboptimal solutions. It is even harder to solve the RSA problem optimally.

One simple yet effective method is to serve the connection requests sequentially in a particular order [33]. Each time a new connection request is served, we first calculate k shortest candidate paths[3] for that connection. Since there are already-served connections existing in the network, the minimum allowable start spectrum slots of the candidate paths are different. We select the path that minimizes the index of the maximum spectrum slots used in the whole network to serve the current connection request, and then move on to process the next one. This is a greedy algorithm that searches for the locally optimal RSA solution that minimizes the estimated spectrum usage for the current connection request.

There are many methods in choosing the order of connection requests. For example, we can order the connection requests according to their requested data rates, and serve first the request with the highest data rate. Alternatively, we can order the connection requests according to the distance from their sources to destinations. Metaheuristics such as the genetic algorithm and simulated annealing have also been applied to optimize the ordering of connection requests [30] at the cost of higher computational complexity.

When solving the RSA problem, the modulation formats of all the channels are not known. We need to use the transmission reaches to estimate the modulation formats. Consequently, the RSA solution is only an approximate estimation of the spectrum usage in the network. In the following transmission parameter optimization stage, fine tuning of the modulation formats is needed to further minimize the spectrum usage as well as guarantee the channel QoTs.

[3]Usually $k = 12$ is sufficient to yield good solutions.

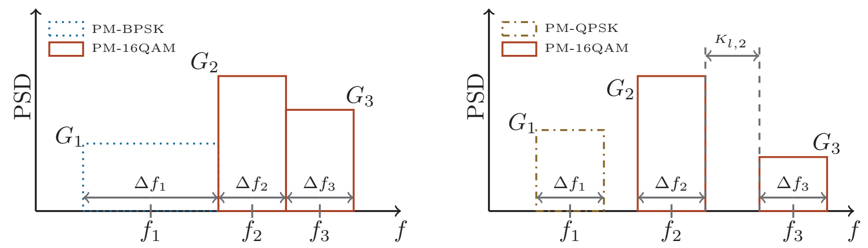

Figure 3.1: Illustration of transmission parameters in flexible-grid optical networks. f_i: the carrier frequency; G_i: transmitted PSD; Δf_i: bandwidth for $i = \{1, 2, 3\}$. The ordering of channels in the spectrum is fixed, but their locations can be changed by choosing different carrier frequencies and modulation formats. $K_{l,2}$ denotes the frequency interval between channels 2 and 3 on this link l.

3.2.2 Transmission Parameter Optimization

The RSA problem solves how the connection requests are routed through the network. If two connections share one common link, the RSA problem also determines their ordering in spectrum domain, which is shown in Figure 3.1. However, since the modulation formats of the connections are not yet selected, it is still necessary to optimize the spectral location of channels. This adjustment is implemented in the transmission parameter optimization stage, where we allocate resources such as modulation formats, PSDs, and carrier frequencies of channels. The aim of this optimization is to minimize the spectrum usage of the connections while guaranteeing the QoTs.

The problem of allocating transmission parameters can be stated as follows:

- *Input*: topology of the network, the traffic matrix, and allocated routes and ordering in spectrum domain for all connection requests
- *Output*: modulation format, PSD, and carrier frequency for each connection request
- *Constraint*: nonoverlapping spectrum constraint and quality of transmission constraint
- *Objective*: minimize spectrum usage in the network, minimize the total transmit power, and maximize total and minimum SNR margins

The primary goal of the transmission parameter optimization is to minimize the spectrum usage of the network. The secondary goal is to maximize the SNR margins of all the connections. The transmitted optical power has the least priority.

We formulate a mixed integer nonlinear programming (MINLP) problem to solve the problem stated above. The auxiliary parameters used in the MINLP formulation are listed below:

- D: set of connection requests

- $|D|$: number of connection requests
- R_i: data rate required by the connection i for $i \in D$
- $\mathbf{R}$: vector of R_i, $\mathbf{R} = \{R_1, \ldots, R_{|D|}\}$
- θ_j: non-negative weighting factors for prioritizing different objectives, $j = \{1, 2, 3, 4\}$

Sets of decision variables in the MINLP formulation are defined as follows:

- $\mathbf{c}$: vectorized continuous variables representing modulation formats of all the channels, $\mathbf{c} = (c_1, \ldots, c_{|D|})^T$
- $\mathbf{f}$: vectorized continuous variables representing carrier frequencies of all the channels, $\mathbf{f} = (f_1, \ldots, f_{|D|})^T$
- $\mathbf{G}$: vectorized continuous variables representing PSDs of all the channels, $\mathbf{G} = (G_1, \ldots, G_{|D|})^T$
- u: maximum spectrum used in the network
- $\mathbf{t}$: vectorized continuous variables representing reciprocals of linear scale SNR margins of all the channels, $\mathbf{t} = (t_1, \ldots, t_{|D|})^T$
- $t_{\min}$: the minimum entry in $\mathbf{t}$

The MINLP formulation for transmission parameter optimization can be expressed as [Paper C]:

$$\begin{aligned} \underset{\mathbf{c},\mathbf{f},\mathbf{G},u,\mathbf{t},t_{\min}}{\text{minimize}} \quad & \theta_1 u + \theta_2 \sum_{i \in D} \frac{R_i}{c_i} G_i + \theta_3 t_{\min} + \theta_4 \sum_{i \in D} t_i && (3.13) \\ \text{subject to} \quad & t_i\, \mathrm{SNR}_i(\mathbf{c}, \mathbf{f}, \mathbf{G}) \geq \mathrm{SNR}_{\mathrm{th}}(c_i), \quad \forall i \in D && (3.14) \\ & \mathbf{K}_l(\mathbf{c}, \mathbf{f}) \leq 0, \quad \forall l \in E && (3.15) \\ & u \geq \frac{R_i}{2c_i} + f_i, \quad \forall i \in D && (3.16) \\ & t_{\min} \geq t_i, \quad \forall i \in D. && (3.17) \end{aligned}$$

Expression (3.13) balances multiple objectives with different weighting factors. Expression (3.14) ensures that the QoTs are satisfied and calculates the inverse SNR margin for each connection. The SNR margin is defined as $t_i = \mathrm{SNR}_{\mathrm{th}}/\mathrm{SNR}_i$, and the inequality (3.14) will be tight because $\sum_{i \in D} t_i$ is minimized in the objective. Inequality (3.15) imposes the nonoverlapping spectrum constraint. Inequalities (3.16) and (3.17) calculate the maximum spectrum usage and the minimum SNR margin in the network, respectively.

In (3.14), the left-hand side calculates the actual SNR for each connection according to the GN model (2.18), whereas the right-hand side takes values from Table 2.1. The function $\mathbf{K}_l(\mathbf{c}, \mathbf{f}) = (K_{l,1}(\mathbf{c}, \mathbf{f}), \ldots, K_{l,|D_l|-1}(\mathbf{c}, \mathbf{f}))^T$ for $l \in E$ is [Paper C]

$$K_{l,j}(\mathbf{c}, \mathbf{f}) = \frac{1}{2} \left(\frac{R_{i_j^l}}{c_{i_j^l}} + \frac{R_{i_{j+1}^l}}{c_{i_{j+1}^l}} \right) + f_{i_j^l} - f_{i_{j+1}^l}. \tag{3.18}$$

Here i_j^l is the j^{th} channel on link l, $K_{l,j}(\mathbf{c}, \mathbf{f})$ denotes the spectrum interval between two neighboring channels (an example is shown in Figure 3.1), D_l is the set of connections using link l, and $|D_l|$ is the number of elements in D_l.

The multiple objectives in the MINLP formulation are balanced by the weighting factors θ_j for $j = \{1, 2, 3, 4\}$. We can flexibly adjust the relative values of the weights to emphasize different goals. First, the values of θ_j should be chosen such that the four terms in the objective, i.e., $\theta_1 u$, $\theta_2 \sum_{i \in D} \frac{R_i}{c_i} G_i$, $\theta_3 t_{\min}$, and $\theta_4 \sum_{i \in D} t_i$, have comparable values. Then, we can assign a higher value to the weighting factor associated with the objective that we want to prioritize. This will alter the optimization direction towards the desired objective.

The numbers of variables in $\mathbf{c}, \mathbf{f}, \mathbf{G}$ and $\mathbf{t}$ are all $|D|$, so the total number of variables is $O(|D|)$. The numbers of constraints in (3.14), (3.16), and (3.17) are all $|D|$. The number of constraints in (3.15) is

$$|E| \sum_{l \in E} |D_l| \leq |E| \sum_{l \in E} |D| = |E|^2 |D|, \tag{3.19}$$

because the number of channels on link l, $|D_l|$, is less than the total number of connection requests. As a result, the total number of constraints is $O(|E|^2|D|)$. The number of variables and constraints are significantly fewer than the previously mentioned ILP formulations. Furthermore, the scalability of the transmission parameter optimization formulation is demonstrated in ring networks [Paper C].

Chapter 4

Contributions

This thesis studies the resource allocation algorithms in flexible-grid optical networks in order to improve the overall spectrum efficiency of the network. In the following, we list the appended papers and summarize their contributions.

1. **Paper A: "Link-Level Resource Allocation for Flexible-Grid Nonlinear Fiber-Optic Communication Systems"**

   In this paper, we propose an optimization formulation to allocate the transmission parameters of all the channels in a single flexible-grid link. The GN model is used in our algorithm to accurately predict the channel QoTs. Compared with a simpler algorithm based on transmission reach, the GN model based optimization can flexibly adjust the channel spacings and modulation formats such that the overall spectrum usage of the system is significantly reduced. In the single link scenario, the spectrum usage is demonstrated insensitive to the spectral ordering of channels.

2. **Paper B: "Resource Allocation in Nonlinear Flexible-Grid Fiber-Optic Networks"**

   In this paper, we describe a novel method for joint optimization of transmission parameters in flexible-grid networks based on the GN model. The channel PSDs, modulation formats, and carrier frequencies are optimized to minimize the spectrum usage and maximize the SNR margins of all the channels. The effectiveness of the proposed algorithm is demonstrated in a simple three-node network. The transmission distances of the optical communication system are also extended with the optimized transmission parameters.

3. **Paper C: "Resource Allocation for Flexible-Grid Optical Networks With Nonlinear Channel Model"**

   This paper extends the transmission parameter optimization in Paper B to more complex network topologies to demonstrate its performance and scalability. We also analyze the relation between modulation formats and transmission distance based on the results of the proposed method. Contrary to the traditional transmission reach method, which assigns a specific modulation format to a given data rate and light path length, the optimal modulation format is actually selected from a range of available modulations.

4. **Paper D: "Power Optimization in Nonlinear Flexible-Grid Optical Networks"**

   In the previous papers, we assume the routes and spectrum orderings of connections are precalculated. In this paper, we develop a mixed integer linear programming formulation to incorporate the routing and spectrum assignment into the resource allocation algorithm. The joint optimization of routes and various transmission parameters can further improve the spectrum efficiency of the network.

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Part II

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